

18. ARO 14919.8-M

G RESEARCH ON THE INVERSE SCATTERING PROBLEM

FOR THE

THREE-DIMENSIONAL SCHROEDINGER EQUATION.

(12)

10 Harry E. Moses

Final Report.

(Period of Grant 1 oct 77-30 September 1 80

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175

U.S. ARMY RESEARCH OFFICE

-411 CAR

Grant No. DAAG29-78-G-0003
Project No. 14919-M

UNIVERSITY OF LOWELL CENTER FOR ATMOSPHERIC RESEARCH 450 Aiken Street Lowell, Massachusetts 01854

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81 2 18

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UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Date Entered)

REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 2. GOVT ACCESSION NO AD-A095 471	3. RECIPIENT'S CATALOG NUMBER
A. TITLE (and Subtitle) RESEARCH ON THE INVERSE SCATTERING PROBLEM FOR THE THREE-DIMENSIONAL SCHROEDINGER EQUATION	5. TYPE OF REPORT & PERIOD COVERED Final Oct. 1977-Sept. 1980 6. PERFORMING ORG. REPORT NUMBER ULRF-411/CAR
7. AUTHOR(s) Harry E. Moses	B. CONTRACT OR GRANT NUMBER(*) DAAG29-78-G-0003
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Lowell, Center for Atmospheric Research, 450 Aiken Street, Lowell, Massachusetts 01854	Project No. 14919-M
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Research Office P. O. Box 12211 Research Triangle Park, NC 27709 14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)	12. REPORT DATE December 15, 1980 13. NUMBER OF PAGES 4 15. SECURITY CLASS. (of this report)
16. DISTRIBUTION STATEMENT (of this Report)	15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

The view, opinions, and/or findings contained in this report are those of the author and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Inverse problem of scattering

Inverse spectral theory for the three-dimensional problem Gelfand-Levitan equation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

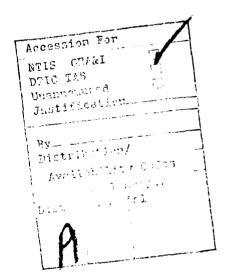
The work performed under the grant is summarize .

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

TABLE OF CONTENTS

		Page
1.	Personnel Supported by the Grant	1
2.	Direction of Research. Accomplishments.	1
3.	Papers Published or Accepted for Publication	2
4.	Papers which are Being Prepared for Submission	2



Personnel Supported by the Grant

- 1. H. E. Moses, Principal Investigator.
- 2. G. Kaiser, Assistant Professor of Mathematics, University of Lowell, a nominal amount of support (less than \$4,000).
- 3. J. M. Cohen, Associate Professor of Physics, University of Pennsylvania, a nominal amount of support (less than \$5,000).

2. Direction of Research. Accomplishments.

The Principal Investigator's principal research direction has been to find appropriate triangularity conditions for the Gelfand-Levitan kernel in three-dimensions. The Gelfand-Levitan kernel for the one-dimensional problem has served as a model.

In this program we have proposed candidate for the Gelfand-Levitan kernel which we hope will give rise to local scattering potentials. A Jost wave function with appropriate completeness properties was given. Green's functions of a Jost type were also given.

We then broadened our attention to the consideration of non-local potentials in three dimensions. By introducing appropriate triangularity properties, we were able to obtain explicit non-local potentials for which the Schroedinger equation could be solved for exactly. A rich spectral theory results from the use of such potentials. Some have very unusual scattering properties, namely they do not scatter at all and thus could not be detected by a scattering experiment. These non-local potentials are the first potentials, non-local or local, which led to simple closed form solutions of the Schroedinger equations (energy-dependent potentials used by earlier workers are not really potentials, strictly speaking).

To understand the nature of triangularity better we considered the one-dimensional analogue of the non-local three-dimensional potentials. The potentials so obtained are the most general parity-dependent potentials and have very interesting scattering properties. These potentials may have technical applications. We are continuing to look into this matter. It was possible to show that a local and a non-local potential could give rise to the same scattering operator and point spectrum.

3. Papers Published or Accepted for Publication

- 1. "Jost Solutions and Green's Functions for the Three-Dimensional Schroedinger Equation," J. Math. Phys., 20, 1151 (1979).
- 2. "A Kernel of Gelfand-Levitan Type for the Three-Dimensional Schroedinger Equation," J. Math. Phys., 21, (1980).
- 3. "Exact Solutions for the Three-Dimensional Schroedinger Equation with Quasi-Local Potentials Obtained from a Three-Dimensional Gelfand-Levitan Equation. Examples of Totally Reflectionless Scattering," Mathematical Methods and Applications of Scattering Theory, Proceeding, Washington, D. C. 1979, Lecture Notes in Physics, Vol. 130, Springer Verlag (J. A. DeSanto, A. W. Saenz, and W. W. Zachary, Editors).
- "An Explicit Example of a Local and a Non-Local Potential whose Hamiltonians are Unitarily Equivalent and whose Scattering Operators are Identical," accepted for rullication by Studies in Applied Mathematics (with P. B. Abraham and B. DeFacio).

4. Papers which are Being Prepared for Submission

1. "Parity-Dependent Potentials in One Dimension," with P. B. Abraham and B. DeFacio.

